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We present a non-local construction of universal gates by means of holonomic (geometric) quantum teleportation. The effect of the errors from an imperfect control of the classical parameters, the looping variation of which builds up holonomic gates, is investigated. Additionally, the influence of quantum decoherence on holonomic teleportation used as a computational primitive is studied. Advantages of the holonomic implementation with respect to control errors and dissipation are presented.

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Holonomic quantum computation (HQC) is a mathematically fascinating area where elements from differential geometry are employed in order to describe the logical evolution of a quantum system with degenerate (multiple) energy eigen-states [1–3]. Recent works [4–6] support the belief that holonomic implementation in NMR, quantum optics or ion traps may be a possible avenue for quantum computation. Here our aim is twofold: first, the scope of HQC is extended to the general framework of universal quantum computation with the construction of non-local gates; second, non ideal HQC implementations, in the form of geometrical imperfections of adiabatic parametric closed paths needed for the construction of holonomic gates, as well as the effect of the simultaneous presence of decoherence, is formulated and investigated.

Specifically, the methodology for the generation of holonomies is as follows. In the control parametric manifold of iso-spectral transformations of a given degenerate Hamiltonian closed paths (loops) are run adiabatically in order to represent the evolution operation for each degenerate eigenspace as a holonomy of a given connection, A [7–9]. The latter has a form defined from the structure of the bundle of the energy degenerate spaces [10]. These loops, when subject to imperfections while spanned, introduce an error in the final gates through their accordingly fluctuated parameters. This is systematically studied for a universal set of gates, consisting of the Hadamard and the control-not (CN) gates, the construction of which is made non locally, by using the teleportation circuit as a computational primitive. As a figure of merit of the effect of both geometric imperfections and quantum dissipation on the overall performance of the teleportation circuit its fidelity is investigated for various limiting values of the decoherence and the imperfection parameters.

As a starting point we shall use the $\mathbf{CP}^n$ holonomic model [10] in order to acquire the desired quantum gates [11]. In [10] a mathematical way for the holonomic construction of given gates by running specific loops in the control parametric manifold is presented. The initial Hamiltonian of this model is given by $H_0 = \varepsilon_0|n+1\rangle\langle n+1|$ which acts on the state-space spanned by $\{|\alpha\rangle\}_{\alpha=1}^{n+1}$. We assume that it is possible by external control to perform equivalent transformations of H_0 given by $\mathcal{O}(H_0) := \{\mathcal{U}\, H_0\, \mathcal{U}^\dagger \,/\, \mathcal{U} \in U(n+1)\}$. Their parametric space is isomorphic to the n-dimensional complex projective space $\mathbf{CP}^n \cong U(n+1)/(U(n)\times U(1))$ which is at the disposal of the experimenter. Each point, $\mathbf{z}$, of the $2n$ dimensional $\mathbf{CP}^n$ manifold corresponds to a unitary matrix $\mathcal{U}(\mathbf{z}) = U_1(z_1)U_2(z_2)...U_n(z_n)$, where $U_\alpha(z_\alpha) = \exp[G_\alpha(z_\alpha)]$ with $G_\alpha(z_\alpha) = z_\alpha|\alpha\rangle\langle n+1| - \bar{z}_\alpha|n+1\rangle\langle\alpha|$ and $z_\alpha = \theta_\alpha e^{i\phi_\alpha}$, for $\alpha = 1,...,n$. If $|\psi\rangle_{in}$ is the initial state in the zero degenerate space of the Hamiltonian, at the end of the adiabatic run of a loop C in the control manifold $\mathbf{CP}^n$ one obtains $|\psi\rangle_{out} = \Gamma_A(C)\,|\psi\rangle_{in}$. The holonomy $\Gamma_A(C) \in U(n)$ has a geometric origin and its appearance accounts for the non-trivial curvature of the bundle of eigenspaces over $\mathbf{CP}^n$. By introducing the Wilczek-Zee connection [8] $A^\mu_{\bar{\alpha}\alpha} := \langle\bar{\alpha}|\mathcal{U}^\dagger(\mathbf{z})\,\frac{\partial}{\partial \mathbf{z}_\mu}\,\mathcal{U}(\mathbf{z})|\alpha\rangle$, with $\alpha, \bar{\alpha} = 1,...,n$, one finds $\Gamma_A(C) = \mathbf{P}\exp\int_C A$ [7], where $\mathbf{P}$ denotes path ordering.

For particular loops the following holonomies are calculated [10]; for the loop $C_1 \in (\theta_\beta,\phi_\beta)$, we obtain an abelian like holonomy, $\Gamma_A(C_1) = e^{-i\Sigma_1}|\beta\rangle\langle\beta| + |\beta^\perp\rangle\langle\beta^\perp|$, where $\mathcal{H} = \text{span}\{|\beta\rangle, |\beta^\perp\rangle\}$. The area $\Sigma_1 =$

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$\int_{D(C_1)} d\theta_\beta d\phi_\beta \cos\theta_\beta$ may be represented as the one of the surface enclosed by C_1 on a S^2 sphere with coordinates $(2\theta_\beta, \phi_\beta)$, while $D(C_1)$ is the enclosed surface on the $(\theta_\beta, \phi_\beta)$ plane. For $C_2 \in (\theta_{\beta}, \phi_{\bar{\beta}})$, $\bar{\beta} > \beta$, we take $\Gamma_A(C_2) = e^{i\Sigma_2}|\bar{\beta}\rangle\langle\bar{\beta}| + |\bar{\beta}^\perp\rangle\langle\bar{\beta}^\perp|$ which is of similar abelian nature as $\Gamma_A(C_1)$. In order to obtain a non-abelian holonomy we perform the loop C_3 on the plane $(\theta_\beta, \theta_{\bar{\beta}})$ positioned at $\phi_\beta = \phi_{\bar{\beta}} = 0$, resulting to $\Gamma_A(C_3) = \exp[-i(-i|\beta\rangle\langle\bar{\beta}|+i|\bar{\beta}\rangle\langle\beta|)\tilde{\Sigma}_3]$, while by taking the plane $(\theta_\beta, \theta_{\bar{\beta}})$ to be at the position $\phi_\beta = \pi/2$ and $\phi_{\bar{\beta}} = 0$, we obtain $\Gamma_A(C_4) = \exp[-i(|\beta\rangle\langle\bar{\beta}| + |\bar{\beta}\rangle\langle\beta|)\tilde{\Sigma}_4]$ where $\tilde{\Sigma} = \int_{D(C)} d\theta_\beta d\theta_{\bar{\beta}} \cos\theta_{\bar{\beta}}$. The identity action on the rest of the states is implied.

With these control manipulations holonomies are produced, which can be used as logical gates with parameters the areas Σ. In fact we obtain a whole set of closed paths in the parametric manifold which give the same holonomies, as deformations of the loop shape and position give the same gate provided their enclosed area is preserved. It is worth noticing that the composition rules [1] of loops of multiplied holonomies may reduce the total length of the transversed paths by combining loops on the same or perpendicular planes for successive gates eventually reducing the required resources for the overall circuit.

Imperfect holonomies. In order to study the errors introduced by imperfect control of the external parameters we adopt an imperfectly spanned loop, C'. If the errors are statistical rather than systematical then the area spanned by this loop are, to the first order, zero. Let us consider how systematic errors in the area effect one and two qubit gates. The Hadamard gate is given by $U_H = \begin{bmatrix} \cos\Sigma & \sin\Sigma \\ \sin\Sigma & -\cos\Sigma \end{bmatrix}$, for $\Sigma = \pi/4$. Up to a corrective phase given by $\Gamma_A(C_1)$ with $\Sigma_1 = \pi$ it may be produced by a loop C_3, with spanning area given by $\Sigma = \int_{D(C_3)} d\theta_1 d\theta_2 \cos\theta_1$ with $D(C_3)$ taken to be a rectangular surface enclosed by $\{0 \leq \theta_1 \leq \pi/2, 0 \leq \theta_2 \leq \pi/4\}$. Introduce an error in this surface by translating the boarders of θ_1 and θ_2 by α and β respectively, where $\alpha, \beta \ll 1$. This is a kind of systematic error. The imperfect Hadamard gate is given to the first order in ε by $U_H(\varepsilon) = U_H + \varepsilon h$, with $h = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$. The integrand $\cos\theta_1$ in Σ is the (θ_1, θ_2) dependent part of the component of the field strength, F, of the connection A, on the (θ_1, θ_2) plane, which is given by $F_{\theta_1\theta_2} = \partial_{\theta_1} A_{\theta_2} - \partial_{\theta_2} A_{\theta_1} + [A_{\theta_1}, A_{\theta_2}]$. As another interpretation of the holonomy, within this approach, is the exponential of the flux of F, [12] then we want this flux to be stable with respect to small deformations of the relevant surface. Hence, we can take this surface to be such that fluctuations of its area give insignificant variations to the total flux. Indeed, the flux enclosed by C_3' is given by $\Sigma(\varepsilon) \approx \frac{\pi}{4} + \varepsilon$, times the Pauli matrix σ_2, with infinitesimal deviation $\varepsilon = \beta$, where the infinitesimal α does not appear at all in the first order, due to the choice of the position of the rectangular's sides. Ideally, we would like $F_{\theta_1\theta_2}$ to be exponentially decreasing with respect to the distance from a particular point of the control manifold so that for large loops centered at that point local deformations of the loop shape would not alter the enclosed flux. A model with such characteristics may be build with optical devices. Indeed, in [5] the one qubit gates $\Gamma_A(C_I) = \exp -i\hat{\sigma}_1\Sigma_I$ with $C_I \in (x, r_1)_{\theta_1=0}$ and $\Sigma_I := \int_{\Sigma(C_I)} dx dr_1 2e^{-2r_1}$ as well as $\Gamma_A(C_{II}) = \exp -i\hat{\sigma}_2\Sigma_{II}$ with $C_{II} \in (y, r_1)_{\theta_1=\pi}$ and $\Sigma_{II} := \int_{\Sigma(C_{II})} dy dr_1 2e^{-2r_1}$, which can produce any one qubit operation, give for large values of the squeezing parameter r_1 zero error in all orders of the loop deformation along r_1. This can be considered as an initial point for passing from geometrical QC to topological QC [13,14]. Further study is needed for the construction of optical (bosonic) two qubit gates with topological character.

On the other hand, the control-not gate is given, up to phase corrections again by a C_3 loop between the proper $(\theta_\beta, \theta_{\bar{\beta}})$ variables with $\Sigma = \pi/2$. By inserting the errors (α', β') in the corresponding components the area becomes $\Sigma(\delta) \approx \frac{\pi}{2} + \delta$, with $\delta = \beta' \ll 1$ the error deviation. Hence, $U_{CN}(\delta) = U_{CN} - i\delta|1\rangle\langle 1| \otimes \mathbf{1}$.

Teleportation Circuit. Consider the teleportation circuit [15] of Fig. 1.

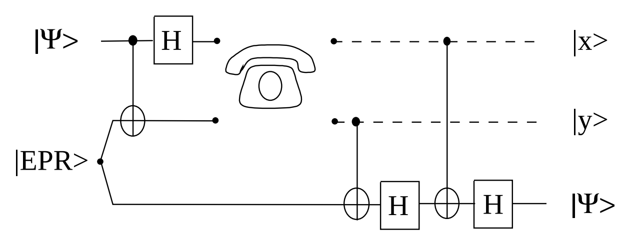

FIG. 1. The Brassard et.al. teleportation circuit. Dashed lines represent classical channels.

Depicted are the unknown state $|\Psi\rangle = a_0|0\rangle + a_1|1\rangle$ which we wish to teleport, the initially employed EPR states $|EPR\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, while the classical states $|x\rangle$ or $|y\rangle$ are the outcomes of measurement in the middle of the circuit. For a more realistic implementation we may use the imperfect Hadamard and CN gates as presented above, where we can take pertubatively into account the imperfections of the Alice and Bob holonomies in order to estimate the reduction of the optimal fidelity of the above scheme, due to the presence of the imperfect holonomic implementation. The overall circuit is represented as $\mathcal{U}_{tel}(\varepsilon, \delta) = \mathcal{U}_6(\varepsilon)\mathcal{U}_5(\delta)\mathcal{U}_4(\varepsilon)\mathcal{U}_3(\delta)\mathcal{U}_2(\varepsilon)\mathcal{U}_1(\delta)$, where, ε is the error for the Hadamard gate and δ is the error for the CN gate. From the six gates presented in Fig. 1, the first CN gate is written as

$$\mathcal{U}_1(\delta) = U_{CN}(\delta) \otimes \mathbf{1} \approx U_{CN} \otimes \mathbf{1} - i\delta|1\rangle\langle 1| \otimes \mathbf{1} \otimes \mathbf{1}$$

$$\equiv \mathcal{U}_1 + \delta V_1$$

which is a unitary matrix up to order $\mathcal{O}(\varepsilon)$. The rest CN's are written similarly. The first Hadamard gate is given by

$$\mathcal{U}_2(\varepsilon) = U_H(\varepsilon) \otimes \mathbf{1} \otimes \mathbf{1} \approx U_H \otimes \mathbf{1} \otimes \mathbf{1} + h \otimes \mathbf{1} \otimes \mathbf{1}$$

$$\equiv \mathcal{U}_2 + \varepsilon V_2$$

and equivalently for $\mathcal{U}_4(\varepsilon)$ and $\mathcal{U}_6(\varepsilon)$. The overall circuit, up to the first order in the error ε or δ is given by $\mathcal{U}_{tel}(\delta,\varepsilon) = \mathcal{U}_{tel} + \delta(V_1 + V_3 + V_5) + \varepsilon(V_2 + V_4 + V_6) \equiv \mathcal{U}_{tel} + \delta V_\delta + \varepsilon V_\varepsilon$. To quantify the error of the teleported state due to the imperfections in the loop spanning we introduce the fidelity

$$\mathcal{F}^{xy}_{\delta,\varepsilon} = \min_{|\Psi\rangle} |\langle xy\psi|\mathcal{U}_{tel}(\delta,\varepsilon)|\Psi EPR\rangle|^2$$

where $|xy\rangle$ is one of the possible outcomes $|00\rangle$, $|01\rangle$, $|10\rangle$ or $|11\rangle$ for the two first qubits due to measurement. In particular we find after minimization and tracing the different possibilities of $|xy\rangle$ the result

$$\mathcal{F}_{\delta,\varepsilon} = 1 - \varepsilon\frac{3}{2}(\sqrt{2}-1) - \delta\frac{1}{2\sqrt{2}} \tag{1}$$

which is smaller or equal to identity for small positive values of ε and δ.

Application. Let us proceed by adopting the teleportation as a kind of universal computation primitive [16], which can accept proper input state (quantum software) in a given site and output in another site universal quantum gates e.g. Hadamard and CN gates. Our aim is to use the Brassard's et.al. circuit of teleportation in order to produce (teleport) H and CN gates. For that we consider imperfect holonomic realization of these generalized circuits and within first order approximation, evaluate their robustness by obtaining their fidelity. Before the teleportation takes place the Hadamard and CN gates can be manufactured without errors, as they are prepared to act on the EPR states, by classical control gates and Bell basis measurement [16]. Due to the similarity of the operators involved the derived fidelities are closely related with the ones in (1). Indeed, for the Hadamard gate the employed circuit is $U^H_{tel} = \mathbf{1} \otimes \mathbf{1} \otimes U_H \; U_{tel} \, \mathbf{1} \otimes \mathbf{1} \otimes U_H^\dagger$ and its fidelity with respect to the transformed initial and final states is equal to the fidelity of the teleportation circuit itself, $\mathcal{F}^H = \mathcal{F}$. For constructing the CN teleported gate we shall employ two teleportation circuits with the additional permutation operator $\Pi_{13} = \sum_{x,y=0,1} |y\rangle\langle x| \otimes \mathbf{1} \otimes |x\rangle\langle y|$. The circuit of the two teleportations (see Fig. 2) is arranged as $W_{tel} = U_{tel} \otimes \Pi_{13} U_{tel} \Pi_{13}$ and the CN implementation is given by $W^{CN}_{tel} \equiv U^{34}_{CN} W_{tel} \; {U^{34}_{CN}}^\dagger$. By calculating the fidelity of this circuit with respect to the rotated initial and final states we obtain $\mathcal{F}_{CN}(\varepsilon,\delta) = \mathcal{F}(2\varepsilon, 2\delta)$, that is, it has the same functional form as in the case of one teleportation but the errors are now doubled. These results are valid for all orders in ϵ and δ.

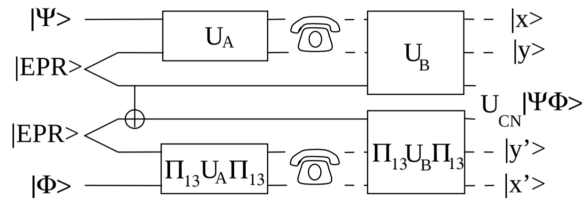

FIG. 2. The CN teleported gate.

Dissipation. Let us now consider the case where imperfections in the construction of holonomic gates as have been studied so far, are present simultaneously with dissipative mechanisms in the modeling of holonomic quantum gates and circuits. As dissipation is an almost unavoidable destruction of quantum coherence that affects the performance quality of logical circuits, it is expected to cooperate with the possible imperfections, in lowering the fidelity of those circuits. To quantify these thoughts we shall formulate the appearance of a general class of dissipative mechanisms in the computational primitive element of an imperfect teleportation circuit. More specifically we shall study decoherence on the Bob's part of the total density operator of the teleportation scheme, that takes place after the completion of Alice's part of the circuit and during the time she classically transmit two bits of information to Bob. This can be also thought of as an imperfection during measurement procedure in the middle of the circuit. Let $\rho_I = |\Psi\rangle\langle\Psi| \otimes \rho$, the initial density operator with $|\Psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$ the state to be transmitted, and ρ the transmitting density operator, which in general can be taken not to be a perfect projector of an EPR entangled pair. Let $\overline{\mathcal{H}} = \mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H}$, the total Hilbert space of Alice and Bob, where $\mathcal{H} = span\{|0\rangle, |1\rangle\}$. Then let $\mathcal{P} = End(\overline{\mathcal{H}})$, the space of pure density operators acting on $\overline{\mathcal{H}}$, and $\mathcal{S} = hull(\mathcal{P})$, the convex hull of $\mathcal{P}$. Consider the linear, trace preserving and completely positive family of maps $\{s_\lambda : \mathcal{S} \to \mathcal{S} : \lambda \geq 0\}$, that admits a Kraus operator-sum representation such that $s_\lambda(\rho) = \sum_{i=1}^k W_i \rho W_i^\dagger$, where $\{W_i\}_{i=1}^k \in End(\overline{\mathcal{H}})$ and $\sum_{i=1}^k W_i W_i^\dagger = \mathbf{1}$. Since we are interested in dissipation occurring in Bob's site only, we take $W_i = \mathbf{1} \otimes \mathbf{1} \otimes V_i$, for some chosen V_i's. Moreover, the dissipation generators $W_i(\lambda)$ may be taken to depend on the parameter λ, in such a way that $\lim_{\lambda \to 0} W_1(\lambda) = \mathbf{1}$, $\lim_{\lambda \to 0} W_i(\lambda) = \mathbf{0}$, $i \neq 1$, namely in the zero dissipation limit $s_{\lambda=0}(\rho) = \rho$. Then we rewrite $s_\lambda(\rho) = \sum_{i=1}^k Ad(W_i)\rho$, where the adjoin action $Ad(X)\rho \equiv X\rho X^\dagger$ is employed. By means of the property $Ad(XY) = Ad(X)Ad(Y)$, we now introduce a 3-parameter POVM $\{\mu_{\delta,\varepsilon,\lambda} : \mathcal{S} \to \mathcal{S} : 0 \leq \delta \leq 1, 0 \leq \varepsilon \leq 1, \lambda \geq 0\}$, where

$$\mu_{\delta,\varepsilon,\lambda}(\rho_I) =$$

$$\sum_{i=1}^{k} Ad(\mathcal{U}_{Bob}(\delta,\varepsilon))Ad(W_i(\lambda))Ad(\mathcal{U}_{Alice}(\delta,\varepsilon))\ \rho_I\ .$$

As in the previous the unitary operators that implement the gates of Alice and Bob in the teleportation circuit are parameterized by the imperfection parameters δ, ε. Next we observe that the dissipation operator on Bob's site commutes with Alice unitary operation i.e. $[\mathcal{U}_{Alice}, W_i] = 0$, $i = \{1, \dots, k\}$, so we have that

$$\mu_{\delta,\varepsilon,\lambda}(\rho_I) = \sum_{i=1}^{k} Ad(\mathcal{U}_{tel}(\delta,\varepsilon))Ad(W_i(\lambda))\rho_I =$$

$$\mathcal{U}_{tel}(\delta,\varepsilon))s_\lambda(\rho_I)\mathcal{U}^\dagger_{tel}(\delta,\varepsilon))\ .$$

At this point there are two ways to proceed. The first one is based on the observation that the above dissipative teleportation scheme is equivalent to the teleportation scheme in which Alice and Bob share a mixed entangled state and an enhancement of the quantum teleportation fidelity is achieved by allowing either of them, to initially perform a local dissipative interaction with the environment [17]. Specifically let $V(.) = \sum_{i=1}^k V_i(.)V_i^\dagger$, then the closeness of the initially shared bipartite state $\mathbf{1} \otimes V(\rho)$, to the ideal maximally entangled state $P_{EPR} = |EPR\rangle\langle EPR|$, is quantified by the *fully entangled fraction* [18], of the bipartite state, $f = max_V Tr(\mathbf{1} \otimes V(\rho) P_{EPR})$. According to the analysis of [17], we search for such V and ρ that $f > 1/2$, so that the optimal fidelity of the teleportation $\mathcal{F} = \frac{2f+1}{3}$, exceeds the limit of the classical communication viz. $\mathcal{F}_{cl} = \frac{2}{3}$.

Alternatively, we can simply proceed by assuming that our dissipative holonomic teleportation has a lower fidelity compared to the ideal teleportation scheme and perform a first order perturbation say, of the holonomic parameters δ, ε in order to estimate how close to one our fidelity can be. Let us take the latter possibility and choose for definiteness the phase damping mechanism described by the $k = 2$ POVM, with $V_1 = diag(1, e^{-\lambda})$, and $V_2 = diag(0, \sqrt{1 - e^{-2\lambda}})$.

In terms of projectors $P_{ab} \equiv |a\rangle\langle b|$, the initial state is written as $\rho_I^\Psi = |\Psi\rangle\langle\Psi| \otimes P_{EPR} = \frac{1}{2}\sum_{i,j\in(0,1)} \alpha_i \overline{\alpha}_j P_{ij} \otimes P_{EPR}$. Similarly if Alice measurement results in two classical bits (x, y), then the final state of teleportation is $\rho_F^\Psi = |xy\Psi\rangle\langle xy\Psi| = \frac{1}{2}\sum_{k,l\in(0,1)} \alpha_k \overline{\alpha}_l P_{xx} \otimes P_{yy} \otimes P_{kl}$. To estimate the quality of the dissipative holonomic teleportation scheme against the standard ideal teleportation we introduce the fidelity factor

$$\mathcal{F}^{x,y}_{\delta,\varepsilon,\lambda} = min_{|\Psi\rangle} Tr(\mu_{\delta,\varepsilon,\lambda}(\rho_I^\Psi)\rho_F^\Psi)\ .$$

After adding up all the different possibilities of x and y we obtain for the fidelity up to the first order in ε and δ, but to all orders in λ the following expression

$$\mathcal{F}_{\delta,\varepsilon,\lambda} = \frac{1}{2}\left(1 + e^{-\lambda}\right) - \varepsilon\frac{3}{2}(\sqrt{2} - 1) + \varepsilon(1 - e^{-\lambda})\left(\frac{21}{32}\sqrt{7} - \frac{51}{32}\right) - \delta\frac{1}{2\sqrt{2}} + \delta(1 - e^{-\lambda})\frac{3}{16}\sqrt{\frac{3}{2}}\ . \quad (2)$$

The intriguing characteristic is that after allowing for dissipation to occur in the initial state by having non-zero values for λ the coefficients of ε and δ become smaller. In particular the coefficient of ε from the value -0.62132 becomes for large dissipation -0.4788, while the coefficient of δ changes from -0.3535 with zero dissipation to the asymptotic value -0.1239. Analytically, let $\Delta\mathcal{F}_{\varepsilon,\delta} = \frac{1}{2} - \varepsilon\frac{1}{32}(21\sqrt{7} - 51) - \delta\frac{1}{4}\sqrt{\frac{3}{2}}$, then expression (2) takes the form $\mathcal{F}_{\delta,\varepsilon,\lambda} = \mathcal{F}_{\varepsilon,\delta} - (1 - e^{-\lambda})\Delta\mathcal{F}_{\varepsilon,\delta}$. Clearly the initial value $\mathcal{F}_{\varepsilon,\delta}$, of the fidelity for zero dissipation $\lambda = 0$, changes to the asymptotic non-zero value $\mathcal{F}_{\delta,\varepsilon,\lambda} = \mathcal{F}_{\varepsilon,\delta} - \Delta\mathcal{F}_{\varepsilon,\delta}$, for large dissipation $\lambda \longrightarrow \infty$. This signifies the fact that the fidelity of imperfect holonomic teleportation becomes resilient to some quantum dissipation that may occur during classical transmission of information. Note that for any $\varepsilon, \delta < 1$ the overall fidelity decreases for increasing values of λ reaching eventually for large dissipation a value greater than 1/2.

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